Question 1, about series network

Consider a 3-station series queueing network with Poisson arrival (rate $\lambda$) and exponential service time (parameters $\mu_1$, $\mu_2$, and $\mu_3$). There is no capacity limit on the queue in front of the first two stations, but at the third station there is a limit of $K$ allowed (including the one in service). If already $K$ customers are in the third station, then any subsequent arrivals to the third station will directly leave the network. Question:

a) Find the expected number of customers in the network (all the 3 stations);

b) Find the expected time spent in the network by a customer who completes all three stages of service.
Question 2, about open Jackson network

Consider an open Jackson network with $M = 4$ servers. The network topology is illustrated as follows. The service rates of servers are $\mu_1 = 1$, $\mu_2 = 1/9$, $\mu_3 = 0.5$, and $\mu_4 = 0.25$.

![Network topology diagram]

Question:

a) Which server is the bottleneck of the whole network?

b) Calculate the minimal value of $\lambda_0$ which makes the utilization ratio of the bottleneck server reach 100% (also call this server is saturated).

c) If we increase the service rate (say $\mu_k$) of the bottleneck server sufficiently, the bottleneck will change from the current server to another different server. Calculate the minimal value of $\mu_k$ which makes such change of bottleneck just happen? Identify which server will become the new bottleneck.

d) If $\lambda_0 = 0.2$, calculate the average value of end-to-end delay of the system.
Question 3, about closed Jackson network

Consider a closed Jackson network with $M = 3$ servers and $N = 4$ customers. The routing probabilities are shown by the following figure. The service rate of server $i$ is denoted as $\mu_i$, $i = 1, 2, 3$ and $\mu_1 = 5, \mu_2 = 4, \mu_3 = 3$.

Question:

a) Assume the relative arrival rate of server 1 is $c$. Calculate the relative arrival rate of other servers in terms of $c$. (1 point)

b) Calculate the normalization constant $G(N)$ using Buzen’s algorithm. (1 point)

c) Calculate the expression of steady state distribution, $P(n_1, n_2, n_3)$. (1 point)

d) Calculate the throughput of the network, i.e., the average number of customers served per unit time. (1 point)

e) Calculate the true value of arrival rate of each server (not in terms of $c$ as question a)). (1 point)

f) Calculate the average response time of each customer. (1 point)

g) Calculate the utilization ratio (or called traffic intensity) of each server. (1 point)

h) Is there a bottleneck in the network? If yes, which is it? (1 point)
i) Assume that the server 1 has a load-dependent service rate $\mu_{1,n_1}$, where $n_1$ is the number of customers at server 1. $\mu_{1,1} = 2, \mu_{1,2} = 3, \mu_{1,3} = 4, \mu_{1,4} = 5$. Calculate $G(N)$ and steady state distribution $P(n_1, n_2, n_3)$. (2 points)