Complete coverage and point coverage in randomly distributed sensor networks

Xi Chen a,∗, Yu-Chi Ho b,a, Hongxing Bai a

a CHNS, Department of Automation, TNList Lab, Tsinghua University, Beijing, 100084, China
b Division of Engineering and Applied Sciences, Harvard University, United States

1. Introduction

Sensor networks which consist of a large number of densely deployed sensors have a wide range of applications such as military sensing, physical security, environment monitoring, traffic surveillance etc. (Akyildiz, Su, Sankarasubramaniyan, & Cayirci, 2002; Chong & Kumar, 2003). When the environment of interest is inaccessible or located in a hostile area, sensors may be air-dropped from an aircraft or by other ways which results in a random placement (Clouqueur, Phitakpanasuphorn, Ramanathan, & Saluja, 2003).

Complete sensing coverage (or complete coverage) and point sensing coverage (or point coverage) are different coverage measures of the sensor networks. Complete coverage means sensor networks can sense the whole area of interest without any vacancy (or hole). In general, complete k-coverage means that any point in the area of interest is covered by at least k, k ≥ 1 sensors. Or there exists no k-vacancy in the area of interest (Hall, 1988). Complete coverage sounds, of course, more reliable than any kind of partial coverage. However, when sensors are deployed in a random way, complete coverage cannot be obtained for sure and the value of its probability becomes important. Basically, point sensing coverage (or point coverage) is one kind of partial coverage. If one point is covered by at least k sensors, then this point is called point k-covered. In general, point k-coverage concerns the portion of the area of interest in which any point is k-covered. Similar to complete coverage, when sensors are randomly distributed, any point in the area of interest cannot be k-covered for sure but with a certain probability.

Complete coverage is ideal and reliable in many situations in which the security (of personnel and/or articles) is of the highest priority. However, partial coverage may be good enough in some applications. For example, if we wish to deploy a sensor network to monitor mountain fire, it is neither possible nor necessary to have the mountainous region being completely covered.

Complete coverage has been well studied. The theory of coverage process (Hall, 1988) provides a basic method for analyzing complete coverage in sensor networks. Using the same decomposition method as presented in Hall (1988) and Zhang and Hou (2005) analyze the required node density for an asymptotical complete k-coverage of the area of interest. They also derive the required sensing range of the sensors in an area of a unit square. By these results, they obtain the upper bound of lifetime when only a partial portion of the area of interest is covered. Kumar, Lai, and Balogh (2004) consider complete coverage with three kinds of deployments in the wireless sensor networks: grid deployment, random uniform, and Poisson. In addition, they assume sensors are active with a probability p. Then they derive the conditions for asymptotic complete coverage of the area of interest under the three deployments respectively. Wan and Yi (2005) assume
that sensors are deployed as either a Poisson point process or a uniform point process in a square or disk region. They take the complicated boundary effect into account and derive the asymptotic requirement of node density for complete $k$-coverage. Different from the above work, to characterize the coverage properties of large-scale randomly distributed sensor networks, Liu and Towsley (2004) define three coverage measures, namely, area coverage (similar to point $1$-coverage in this paper), node coverage (to check the redundancy of a sensor) and detectability (capability of the sensor networks to detect objects moving in the networks). They show that there exists a critical density for object detection.

In this paper, we first derive the formula for the probability of point $k$-coverage. Then, we compare point coverage with complete coverage in details. By analysis and simulation, we reveal the numeric relation between the probabilities of point coverage and those of complete coverage. Lastly, we conclude that, with the additional data about point coverage, network designers can make better network and budget planning.

The rest of the paper is organized as follows. In Section 2, the formulation of the network model is presented. In Section 3, analysis of the probability of point coverage is carried out. Comparison between point coverage and complete coverage is made. Finally, Section 4 concludes the paper.

2. Network model

Consider a sensor networks which consists of $N$ sensors. All sensors are identical in terms of sensing radius, energy as well as any other capabilities. Sensors are independently and identically distributed in the area of interest following uniform distribution. In particular, when the area of interest is assumed a unit square, $N$ also represents sensor density.

Coverage performance is dependent on sensing model of sensors. In this paper, the sensing model is assumed to be a Boolean sensing model. A sensor's sensing range is a disc with the sensor as its center and $r$ as its radius and $s = \pi r^2$ as its sensing area. Moreover, as we consider a densely distributed sensor networks, we assume $s$ is much less than the whole area of interest.

Majority of the work introduced in Section 1 adopted Poisson distribution in analysis and derivation. In fact, Poisson distribution has a close relation to binomial distribution as discussed in Hall (1988). Take the area of interest to be a square with width $l$, and suppose $l$ and $N$ increase together in such a manner that $N/l^2 \rightarrow \lambda$, where $\lambda$ represents the density of sensors and $0 < \lambda < \infty$. Let $S$ be a bounded subset in the area of interest. For a large $l$ the area of interest completely envelops $S$, and then the chance that $S$ contains precisely $j$ of the uniformly distributed sensors equals $C_l j \left( \frac{l}{\lambda} \right)^j \left( 1 - \frac{l}{\lambda} \right)^{l-j}$, which follows binomial distribution $B(n, \frac{l}{\lambda})$, where $S$ represents the area of $S$. As $l$ increase, in the limit, the number of sensors in $S$ is Poisson distributed with mean $\lambda |S|$. Hence, Poisson distribution is an asymptotic approximation of binomial distribution. For the rigorous proof of such approximation, one may refer to Ross (1996).

Concisely and collectively, assumptions imposed in this paper are listed in the following.

Assumption 1. Without loss of generality, the area of interest is an unit square, i.e. its area $A = 1$.

Assumption 2. Sensors are homogeneous.

Assumption 3. Boolean sensing model with sensing radius $r$ is adopted.

Assumption 4. Sensing radius $r$ is much less than $1$.

Assumption 5. Sensors are identically and independently distributed in the area of interest following uniform distribution.

3. Complete coverage vs. point coverage

In this part, we first explain the meanings of complete coverage and point coverage. Then we present the bounds of the probability of complete coverage and derive the probability of point coverage and further explore the different features of complete coverage and point coverage.

3.1. Complete coverage

The implication of complete coverage can be clarified with a simple example. Denote $P_{c1}$ as the probability of complete $1$-coverage. If $P_{c1} = 0.99$, then we have the following interpretation. Suppose we perform 100 independent experiments of randomly distributing the $N$ sensors in the area of interest. Then on the average we will find in 99 experiments, the $N$ sensors will completely cover the whole area. But in one experiment the area of interest will not be completely covered.

Although complete coverage is well studied as we introduced in Section 1, the exact probability of complete coverage is not obtained yet but its bounds are derived in references (e.g. Mo, Qiao, and Wang (2005)).

Theorem 6 (Bounds of Complete Coverage). Suppose $\hat{P}_{ck}$ and $\bar{P}_{ck}$ are the upper bound and lower bound of probability for complete $k$-coverage respectively. Then

\[
\hat{P}_{ck} = \frac{4(k+1)!/(Nps)^{k-1}e^{-Nps}}{1 + 4(k+1)!/(Nps)^{k-1}e^{-Nps}},
\]

\[
\bar{P}_{ck} = 1 - 2e^{-Nps} \left( 1 + (N^2p^2s + 2Npr) \sum_{i=0}^{k-1} \frac{(Nps)^i}{i!} \right),
\]

where $s = \pi r^2$ and $p$ is the probability for a sensor to be active.

To save power and prolong its life, a sensor may switch between states of active and asleep. One common way is to let each sensor be randomly active with a certain probability (e.g. see Chen, Bai, Xia, and Ho (2007) and Ren, Li, Wang, Chen, and Zhang (2005)).

3.2. Point coverage

Based on Assumptions 2–5, we know that each point in the area of interest has the same probability of being $k$-covered. Therefore, this probability is also called as the probability of point $k$-coverage. The implication of point coverage can be illustrated with the following example. Denote $P_{p1}$ as the probability of point $1$-coverage. If $P_{p1} = 0.99$, then, if we arbitrarily or randomly pick out 100 points in the area of interest, on the average, one of these 100 points will not be covered. Or sensors collectively cover 99% portion of the whole area of interest.

If a point is point 1-covered, then there exists at least one sensor located within the distance of $r$ to the point (or falls in the circle with the point as its center and $r$ as its radius). Hence, $P_{p1} = 1 - (1 - \frac{r}{\lambda})^N = 1 - (1 - s)^N$. In general, we denote $P_{pk}$ as the probability of point $k$-coverage. Then we have

\[
P_{pk} = 1 - \sum_{i=0}^{k-1} C_{0}^i (s)^i (1 - s)^{N-i}.
\]

If sensors follow a scheme to be randomly active with a certain probability $p$, we have the following theorem for the probability of point $k$-coverage.

Theorem 7. Suppose $p$ is the probability for a sensor to be active and $P_{pk}(p)$ is the probability for point $k$-coverage. Then

\[
P_{pk}(p) = \sum_{i=0}^{k-1} C_{0}^i (p)^i (1 - p)^{N-i}.
\]
First, we know that and Kumar are derived with binomial distribution.

\[ P_{\text{ak}}(p) = 1 - \sum_{i=0}^{k-1} C_i^N (ps)^i (1 - ps)^{N-i}, \quad k = 1, 2, \ldots \]  

where \( s = \pi r^2 \).

**Proof.** First, we know that

\[ \text{Prob}(\text{There are exactly } n \text{ of } N \text{ sensors being active}) = C_n^N p^n (1 - p)^{N-n}. \]

Since each sensor’s state (active or asleep) is independent of its location and independent of other sensors’ state, we have

\[ \text{Prob}(\text{There are exactly } i \text{ of } n \text{ active sensors in } s) = \sum_{i=0}^{N} C_i^N C_n^N (ps)^i (1 - ps)^{N-i} (1 - p)^{N-n} \]

(set \( l = n - i \))

\[ = \sum_{i=0}^{N-l} C_i^N C_{N-l}^N (ps)^i (1 - ps)^{N-i} (1 - p)^{N-n} \]

(use \( C_i^N = C_{N-i}^N \))

\[ = \sum_{i=0}^{N-l} C_i^N C_{N-l}^N (ps)^i (1 - ps)^{N-i} (1 - p)^{N-n} \]

\[ = \sum_{i=0}^{N-l} C_i^N (ps)^i (1 - ps)^{N-i}. \]

Therefore, the probability for point \( k \)-coverage is

\[ P_{\text{ak}}(p) = 1 - \sum_{i=0}^{k-1} C_i^N (ps)^i (1 - ps)^{N-i}. \]

The proof is completed. \( \square \)

In particular, \( p = 1 \) means that sensors always keep active and in this case, (3) reduces to (2). Please note that, as we have assumed that \( r \ll 1 \) (Assumption 4), boundary effect is neglected in the above derivation. The asymptotic property of point coverage is obvious, that is, when \( N \rightarrow \infty, P_{\text{ak}} \rightarrow 1 \) provided \( p \neq 0 \).

Once \( N, r \) and \( p \) are given, with (3) we can calculate the values of \( P_{1k}, P_{2k}, P_{3k}, \ldots \) and so on. For any fixed \( k, P_{\text{ak}} \) tells that, on the average, how large a portion of the area of interest is \( k \)-covered. \( P_{\text{ak}}(p) \) decreases as \( k \) increases. \( \{P_{\text{ak}} \mid k = 1, 2, \ldots \} \) characterize the coverage performance of the networks more clearly.

Formula (2) and (3) are derived with binomial distribution and they are the exact probability of coverage when sensors are distributed independently and uniformly in the area of interest. As sensors are densely deployed, Poisson distribution is a good approximation of binomial distribution so that Poisson distribution is usually used to analyze complete coverage in the related literatures (see Kumar et al. (2004), Wan and Yi (2005) and Zhang and Hou (2005), etc.). We directly provide the approximation formula for the probability of point \( k \)-coverage but omit the details of its derivation. Denote \( P_{\text{ak}} \) as the approximate probability of point \( k \)-coverage with Poisson distribution. Then

\[ P_{\text{ak}}(p) = \text{Prob}(\text{There are at least } k \text{ active sensors in } s) \]

\[ = 1 - \sum_{i=0}^{k-1} \frac{e^{-Nps}(Nps)^i}{i!}, \quad k = 1, 2, \ldots \]  

3.3. Comparison with theoretical results

After sensors are deployed in the area of interest, the answer to complete coverage is a boolean: ‘Yes’ (the whole area of interest is covered) or ‘No’ (the area of interest is not fully covered) and the answer to point coverage is a real number between 0 and 1 (how much area is covered in proportion to the area of interest). Therefore, complete coverage is much more sensitive to each sensor’s location. Suppose \( N \) sensors have fully covered the area of interest. A small change in any sensor’s location most likely will leave a tiny hole so that complete coverage will not be retained. However, such small change in a sensor’s location can hardly affect the point coverage dramatically.

On one hand, \( P_{1k} \rightarrow 0.99 \) means that 1% of the time the area of interest won’t be completely covered. When not completely covered, we have no safeguard against how much of the area is uncovered. On the other hand, \( P_{1k} = 0.99 \) means that on the average 1% of the points in (or 1% portion of) the area of interest will be uncovered. Probabilities of the point coverage (3) or (4) can reveal, on the average, how much of the area of interest is 1-covered, 2-covered, 3-covered and so on. Such numbers are much more informative than the probability of complete coverage.

First of all, for a sensor network, we can directly observe that

\[ P_{\text{ak}} = \text{Prob}(\text{The area of interest is completely } k\text{-covered}) \]

\[ \leq \text{Prob}(\text{Two points } O_1 \text{ and } O_2 \text{ in it are both } k\text{-covered}) \]

\[ \leq \text{Prob}(\text{Point } O_1 \text{ is covered by } k \text{ sensors}) \]

\[ = P_{\text{ak}}. \]

Thus \( P_{\text{ak}} \leq P_{\text{ak}} \) holds for sure. This is true for any kind of sensor network, even when sensors are not randomly distributed or sensors have different sensing radii. As the exact probability for complete coverage, \( P_{1k} \), is not available, we use its upper bound, \( P_{1k} \), defined in (1), to replace \( P_{1k} \) in the following data comparison. Therefore, the following comparison is rather conservative. So we can compare complete coverage with point coverage in terms of \( N, r \) and \( p \) since both of the probabilities are determined by these three parameters. In Figs. 1 and 2, the star curve represents the upper bound of probability complete 1-coverage, \( P_{1k} \), which is calculated according to (1). The other three curves represent the probabilities of point 3-coverage, point 4-coverage and point 5-coverage respectively according to (4).

Fig. 1 shows that, if \( N = 341, r = 0.1 \) and \( p = 1 \), then complete coverage happens with probability 99% while, on the average, more than 99% portion of the area of interest has been 4-covered and more than 98% portion of the area of interest has been 5-covered.
Fig. 2. $P_{c1}, P_{c2}, P_{c3}$ and $P_{c1}$ vs. $N$ when $r = 0.18$ and $p = 1$.

Fig. 3. $P_{c6}, P_{c7}, P_{c8}$ and $P_{c1}$ vs Number of nodes when $r = 0.1$.

Fig. 4. $P_{c6}, P_{c7}, P_{c8}$ and $P_{c1}$ vs Number of nodes when $r = 0.18$.

The curve of complete coverage is a zigzag. Its fundamental reason is that complete coverage is too sensitive to the locations of sensors as we have discussed at the beginning of Section 3.3. We carry out the simulation with $r = 0.18$ as we do with $r = 0.1$ and plot the results in Fig. 4.

The simulation provides us a more accurate comparison between complete coverage and point coverage. In Fig. 3, the curve of probability for complete coverage $P_{c1}$ falls between $P_{c6}$ and $P_{c10}$ and in Fig. 4, the curve of $P_{c1}$ falls between $P_{c6}$ and $P_{c8}$.

4. Concluding remarks

Although complete coverage and point coverage are different coverage measures of sensor networks, the comparison between them enables us to understand each of them more deeply. For a specific application, no matter which kind of coverage is chosen, observations and results obtained in this paper are helpful for network designers.

Acknowledgements

The authors would like to thank anonymous referees for their valuable comments on the earlier versions of this paper.
The first and third authors were partially supported by the National Natural Science Foundation (60574087, 60574064 and 60736027) of China. The second author was partially supported by ARO contract DAAD19-01-1-0610, AFOSR contract, F49620-01-1-0288 and NSF grant ECS-0323685.

References


Xi Chen received her B.Sc. and M.Eng. from Nankai University, Tianjin, China, in 1986 and 1989, respectively. After graduation, she worked in the Software Engineering Institute at Beijing University of Aeronautics and Astronautics for seven years. From October 1996 she studied in the Chinese University of Hong Kong and received her Ph.D. in 2000. Then she worked as a post-doctoral fellow in Information Communication Institute of Singapore and in the Department of Systems Engineering and Engineering Management, Chinese University of Hong Kong. Since July 2003, she works in the Center for Intelligent and Networked Systems (CFINS), Department of Automation, Tsinghua University, Beijing, China. Her research interests include wireless sensor networks and stochastic control.

Yu-Chi Ho received his undergraduate and graduate education from MIT and Harvard, respectively. For over 40 years he taught and did research on the Harvard faculty. He is active professionally in numerous capacities as editor of journals, author of book and paper classics, and received numerous awards. He is a Life fellow of IEEE, an elected member of the US National Academy of Engineering and an elected foreign member of the Chinese Academy of Sciences and the Chinese Academy of Engineering. Since October 2001, He also acts part time as the chief scientist and chaired professor of the Center for Intelligent and Networked Systems (CHINS) at Tsinghua University, Beijing, China.

Hongxing Bai received his B.E. degree in 2003 and Ph.D. degree in 2008 from Department of Automation, Tsinghua University, Beijing, China. He is currently a project manager at the JoinTown Pharmaceutical Group Co. Ltd. His research interests include wireless sensor networks, operations research, supply chain management, and logistics.

Please cite this article in press as: Chen, X., et al. Complete coverage and point coverage in randomly distributed sensor networks. Automatica (2009), doi:10.1016/j.automatica.2009.02.020